Carlton Academy

Bridging the Gap to A Level Mathematics

Why do you need bridging work?

Welcome to your A Level Maths warm-up. Students who want to continue studying maths at A Level are usually expected to complete a unit of work before they start. This is now more important than ever, to ensure that you have all of the skills required to be able to confidently access the course when you return.

What are our expectations?

Your maths bridging work was given out during your taster lesson. If you missed it for any reason, please contact the school and we will be able to leave you a copy of the booklet at reception. It includes a mixture of tasks which will prepare you for your first main assessment, but they will also encourage some deeper understanding and enjoyment of the topics. The expectation in that this booklet is completed by the time you return in September.

What will you be studying?

The work that you have been set recaps some GCSE topics. If you fully understand the content then some of the tasks will not take much time. If you need some revision then please use either your Hegarty or Dr Frost accounts. There are also numerous useful tutorials on Youtube. At the end of the booklet there is a shadow paper which you can use to prepare yourself for the baseline assessment that you will take to see if mathematics is a suitable A Level for you.

What do you do if you are finding it too easy or difficult?

If you complete the tasks that have been set and would like to prepare yourself further please use the resources provided below. If you are struggling with a topic, or are having trouble accessing any of the resources that you need, please feel free to email me from your **academy email** on s.brenan@theacademycarlton.org.

Interesting Thinking - 10 Olympic Problems

Consider some of these questions concerning the mechanics of sport. You might need more information in some cases or you might need to make an approximation to allow for mathematical modelling. The important thing in this exercise is not your answers, it is your ability to think mathematically and your ability to explain your thought processes. Please email me your solutions at s.brenan@theacademycarlton.org.

1. What if a long jumper could launch themselves from the platform at 45 degrees with the same speed as at their standard launch angle? Would they jump further?

2. In pistol and rifle events, competitors aim at a 10-ringed target from the set distances of 10m, 25m and 50m. Do you think that marksmen need to alter their angle of aim by a measurable amount between these targets?

3. Imagine that a wind of speed $1ms^{-1}$ is blowing parallel to the straight parts of the athletics track. Do you think that this would help or hinder a 400m sprinter?

4. What if a shot-putter could launch the shot at an angle of 45 degrees at the same speed as their usual launch angle?

5. At what speed does a pole-vaulter hit the crash mat?

6. In football, a penalty is taken 12 yards away from the goal. How good do the goalkeeper's reactions have to be?

7. A basketball free throw is taken 4.6m from the hoop. The hoop is 45.7cm in diameter, and 3.05m high. The basketball is 24cm in diameter. How precise does a player's shot have to be to ensure the ball goes in the hoop?

8. A trampolinist can jump to a height of 10m. They perform a double somersault. How quickly must they be able to rotate in order to land safely on their feet and not on their head?

9. A gymnast is swinging on a high bar. The distance between his waist and the bar is 0.90m. At the top of the swing his speed is momentarily $0ms^{-1}$. Calculate his speed at the bottom of the swing.

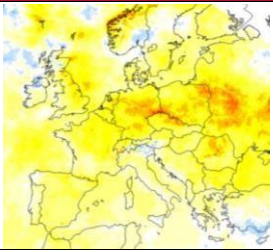
10. Assuming the ball travels at a constant speed throughout, how much longer does a tennis serve to the edge of the court take to reach the baseline than a serve 'down the T'?

SUGGESTED SOLUTIONS CAN BE FOUND AT THE END

Interesting Reading

There are many interesting news articles that include maths. Whether they involve the maths of sport, the use of statistics or how maths can be used to predict the future, they are an interesting and informative way to relate the work that you do in the classroom to every-day life. It could be argued that there has never been a more important time to be a mathematician!

Climate change: 2019 was Europe's warmest year on record



Europe is heating faster than the global average as new data indicates that last year was the warmest on record.
https://www.bbc.co.uk/news/science-environment-52380157

Coronavirus R0: Is this the crucial number?

There is a simple, but crucial number at the heart of understanding the threat posed by the coronavirus. It is guiding governments around the world on the actions they need to take to save lives and it gives us clues to the extent that lockdown can be lifted.
https://www.bbc.co.uk/news/health-52473523

Secret of Usain Bolt's speed unveiled.

Scientists say they can explain Usain Bolt's extraordinary speed with a mathematical model.
https://www.bbc.co.uk/news/science-environmenthttps://www.bbc.co.uk/news/science-environment-2346281523462815

Can you find any interesting articles of your own?

Interesting Watching

There are many mathematical documentaries and films that you can watch. Netflix and Youtube are two of the platforms that have numerous up to date programmes involving maths. There are some suggestions below.

Prediction by the Numbers – Netflix

With the science of forecasting flourishing, this documentary explores how predictions inform our lives and statistics alogorithms' reliability.

The Code – Netflix and Youtube

Mathematics professor Marcus Du Sautoy begins the journey with a look at how significant numbers appear throughout the world, governing all life.
https://www.youtube.com/watch?v=eOMZtBacarY&t=1522s

Dream Big – Netflix and Youtube

This compelling documentary examines some incredible achievements of engineering from across the globe. https://www.youtube.com/watch?v=bCaT-SN_Zuk

Christmas Lectures 2019 Hannah Fry – Youtube

The Royal Institution Christmas Lectures are annual lectures by the Royal Institution presenting scientific subjects the general public in an entertaining manner.
https://www.youtube.com/watch?v=_q4DrUHKC0Q
https://www.youtube.com/watch?v=TtisQ9yZ2zo

10 Olympic Solutions

These are not the "correct solutions". These are thoughts submitted by an A Level student.

1) Ignoring air resistance, if a long jumper could launch themselves at 45° at a speed of 10.0ms^{-1}, they'd travel $\frac{10^2}{g}\text{m} = 10.2\text{m}$. (See here for a derivation). In reality, long jumpers would probably struggle to convert enough of their horizontal speed into the required vertical speed to travel on this trajectory. There's an interesting article here discussing how good a long jumper Usain Bolt could be.

2) Assume the gun is fired from exactly the same height as the target (though any height difference could be incorporated fairly simply into the model), and the bullet travels at a constant speed of $v=400\text{ms}^{-1}$. Suppose the bullet is fired at an angle α to the horizontal. The coordinates of the bullet are

$$\left(v\cos(\alpha)t, v\sin(\alpha)t-\frac{g}{2}t^2\right).$$

At time $t=T$ say, the bullet hits the target. If L is the distance to the target from the shooter, we find $\sin(2\alpha)=\frac{Lg}{v^2}$ (using $\sin(\alpha)=2\cos(\alpha)\sin(\alpha)$.) This gives the following values of α for the different distances:

L (m)	α **(°)**
10	0.035
25	0.088
50	0.18

These seem to be very small numbers. However, if we included air resistance, the bullet would slow down during the motion and consequently hit the target lower than predicted here, so in reality there might need to be a noticably different aim for the different distances.

3) Suppose the athlete is only affected by the wind on the back and home straights, which are around 85m in length, and they run exactly 1ms^{-1} faster with the wind and 1ms^{-1} slower when running against the wind. If the athletes' time in still conditions is 50s, then their average speed during the race is 8ms^{-1}. Their wind affected time using these assumptions would be:

$$\frac{(400-2\times85)\text{m}}{8\text{ms}^{-1}}+\frac{85}{8+1}\text{s}+\frac{85}{8-1}\text{s}=50.3\text{s}$$

i.e. their time is predicted to be 0.3s slower.

We could incorporate other factors into the model. Air resistance, for example, is proportional to the speed squared, so could affect the time significantly. The wind would also affect the athlete around the bends of the track; here the wind strength would vary continuously. We've assumed the wind would affect the speeds along the straights symmetrically, in general this is likely to not be the case, as the athletes'' performance may be reduced more than expected running into the wind.

4) A launch speed of 13ms^{-1} seems a reasonable approximation. Neglecting air resistance, a throw with the velocity angled at 45° to the ground would travel $\frac{13^2}{9.8}\text{m} = 17.2\text{m}$. The speed at which a shot putter can launch the shot depends on the launch angle. For more information, see here.

5) Suppose the athlete has just equalled the world record of 6.14m. Assume the athlete has zero vertical speed at the top of the jump. Their speed at the bottom of the descent is therefore $\sqrt{0^2+2\times9.8\times6.14}\,\text{ms}^{-1}=11.0\text{ms}^{-1}$.

6) A footballer can kick the ball at a speed of around 75mph. Assuming the football remains at this speed, it'll take $11\text{m}\times\frac{1}{75\times\frac{1600}{3600}\text{ms}^{-1}}=0.33\text{s}$ to reach the goal. The goalkeeper needs to have moved into position and be ready to save it in this time!

7) Assume for the time being, the vertical velocity and trajectory result in the ball landing in the centre of the hoop looking from above. In the horizontal plane, the athlete can shoot at an angle α off the centre, where $\alpha=\tan^{-1}\left(\frac{0.23}{4.6}\right)=2.8^\circ$. Can you work out any restrictions on the angle to the horizontal of the initial velocity?

8) Using $s=ut+\frac{1}{2}at^2$ shows that the time (for a point particle) to fall from 10m is $\sqrt{2\times10/9.8}\,\text{s}=1.43\text{s}$. Thus, the time taken to travel to a height of 10m and back is $2\times1.43\text{s}=2.86\text{s}$. If they need to rotate 720° in that time, they need to rotate at 252°s^{-1}.

9) If we assume all his mass is 0.9m from the bar, then conservation of energy gives: $mg(2\times0.9)=\frac{1}{2}mv^2$, so $v\approx6\text{ms}^{-1}$.

10) A 'singles' tennis court measures 8.3m by 23.8m. Suppose the player can serve at 50ms^{-1}. The straight serve takes $\frac{23.8}{50}\text{s}=0.476\text{s}$ to get to the baseline. The diagonal serve travels $\sqrt{4.15^2+23.8^2}\,\text{m}=24.2\text{m}$, so takes 0.49s.